

Model based numerical state feedback control of jet impingement cooling of a steel plate by pole placement technique

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ABSTRACT

This article presents the application of a state feedback, Pole-Placement temperature-tracking control formulation leading to a comprehensive Single-input, multiple-output (SIMO) structure for cooling of a steel plate by an impinging air jet. Following a semi-discrete control volume formulation of the 1-D transient heat conduction; the state space model of nodal variation of metal temperature with time has been arrived. Stable temperature track with respect to the reference temperature track was obtained by using the state feedback control algorithm. To solve this prescribed state feedback problem, an efficient pole placement technique was implemented in two ways i.e., initially, the fixed poles and then variable poles in the complex plane. The control models were simulated in the MATLAB and SIMULINK environments. The state feedback pole placement technique was found efficient in controlling the parameters in the given application. It was observed that the response of the non-linear system is sensitive to linearization time interval. Better control is implemented by increasing the frequency of adjustment of the closed-loop poles.

KEYWORDS: Feedback Control, Jet Impingement Cooling, Pole Placement Technique, State Space Method, Single Input Multiple Output Problem, Fixed Poles, Variable Poles, Temperature Control

Nomenclature:

K: The feedback gain matrix
A: The coefficient matrix in the state space equation
A_i: The coefficient matrix in the state space representation for ith model
The mean coefficient matrix input
X: The matrix of state variables
matrix of mean state variables
λ: The eigen values
ith number of transformable models (s⁻¹)
transformation matrix
Z: The assumed state variable matrix
number of operating conditions

t: Time (s)
A: The coefficient matrix of control input
T₁: Temperature at the top surface of the steel plate (K)
B_i: The coefficient matrix of control input or ith model
X̄: The matrix of state rates
a_c: The perturbation in strain rates
X': The error matrix (X - X̄)
T₁: The demand (K/s)
P_i (S): The
a: The control input (Strain rate)
ā: The mean strain rate (mean control output)
A-BK: The feedback matrix
I: The identity matrix
T: The
r: The

I. INTRODUCTION

Heat treatment involves the method of controlled heating, soaking and cooling temperature cycles that are applied to a material or substrate, to impart superior metallurgical properties to it. Heat treatment has wide application and is most abundantly used in the field of glass, ceramic, iron and steel production industries.

The dynamics of heat transfer systems are highly nonlinear. Irrespective of nonlinearity in of heat transfer systems, air jet impingement cooling systems also have model uncertainties which includes large changes in the air flow rate and variation of parameters, the external disturbances, leakages, and friction. To overcome this problem in order to gain profits, process designers are creating designs and modelling in complex non-linearity regions where process controllers can face stiff the challenges. In model predictive control, the controller action is the solution to a constrained optimization problem that is solved numerically on-line [1, 2, 3]. In contrast, differential geometric control is a direct synthesis approach in which the controller is derived by requesting a desired closed loop response in the absence of input constraints [4, 5]. In Lyapunov-based control, closed loop stability plays a central basic role in a controller design. [6, 7, 8]. Kravaris and Daoutidis [9] presented a non-linear state feedback controller for second order non-linear systems. Niemiec and Kravaris [10] proposed a systematic procedure for the construction of statically equivalent outputs with prescribed transmission zeros. Kanter et al. [11] developed nonlinear control laws for input constrained, multiple-input, multiple-output, stable processes. They addressed the nonlinear control of the processes by exploiting the connections between model- predictive control and input-output linearization. Kravaris et al. [12] presented a systematic method arbitrarily assigning the zero dynamics of a nonlinear system by constructing the requisite synthetic output maps. The method requires solving a system of first-order, nonlinear, singular PDEs. Mickle et al. [13] developed a tracking controller for unstable, nonlinear processes by using trajectory linearization. Tomlin and Sastry [14] derived tracking control laws for non-linear systems with both fast and slow, possibly unstable, zero dynamics. Vander Scaft [15] developed a nonlinear state feedback H_∞ optimal controller. Devasia et al. [16] and Devasia [17] introduced an inversion procedure for nonlinear systems that constructs a bounded input trajectory in the pre-image of a desired output trajectory. Hunt and Meyer [18] showed that under appropriate assumptions the bounded solution of the partial differential equation of Isidori and Byrnes [19] for each trajectory of an ecosystem must be given by an integral representation formula of Devasia et al. [17] Chen and Paden [20] studied the stable inversion of nonlinear systems.

The present work uses the same continuous-time, model predictive control framework. Conceptually as the amount of nonlinearity is very less so the model does not require any linearization of the process equations. Here the nonlinear terms are considered as the strain rate in the convective heat transfer correlation. The controller introduced here is obtained by requesting desired responses for the state variables. The nonlinear state feedback is derived by minimizing a function norm of the deviations of the controlled outputs from linear reference trajectories with orders equal to the (output) relative orders. In the part of the work described in this chapter, we come up through a systematic evaluation with a process model that provides the starting point of evolving a model-based control. At the same time, we analyze the performance of a model-based nonlinear control with the applications of an efficient pole-placement technique. The focal point of the present analysis is a detailed modeling of the heat transfer and applications of efficient techniques (pole-placement) to control the state variables influencing the process. Moreover this chapter presents a nonlinear control method that is applicable to stable and unstable processes. The closed loop stability is ensured by forcing all the process state variables to follow their corresponding reference trajectories. The proposed control system includes a nonlinear state feedback and pole placement technique in both the forms of fixed poles and variable poles.

1.1. Pole placement techniques

“Pole placement” is a conventional design methodology for linear time-invariant control systems. The method is based on the fact that several performance specifications can be met by using dynamic output feedback to adequately placed closed loop poles in the complex plane. . An extension of the classical pole placement problem is the regional pole-placement problem, in which the objective is to place closed-loop poles in a suitable region of the complex plane. In many real cases, the model uncertainty reflects on the parameters of the plant, which has motivated extensive research efforts in parametric robust theory [21, 22, 23]. Pole placement methods can be divided mainly into two categories such as output feedback and state feedback methods. Usually dominant pole design techniques are employed as it is not always possible to assign all the poles of a system and thus one has to relax the design requirements and consider a partial pole placement that will give satisfactory overall performance. The pole placement problem is much easier to handle when the design is done in the state space domain and many techniques exist for this particular case. In this work, the plant is represented by proper state-space equation. Robust pole placement problem is formulated as the problem of robustly assigned poles in a region in two different ways i.e., by assuming some appropriate fixed poles and variable poles in terms of the Eigen values of the ($\mathbf{A} - \mathbf{BK}$) matrix. Where ‘ \mathbf{K} ’ is the gain. Similar conditions hold in [24] and [25].

The main focus of this work was to present a more efficient way of approaching Ackerman's formula to find the gain matrix so as to achieve the Eigen poles. Apart from this, the technique presented here was utilized towards significant progress for the case of nonlinear Single-input and multiple-output (SIMO) systems. . As only the temperatures of the plate are controlled, typically the plate may remain insignificantly cool or the desired cooling temperature trajectory is followed backwards. That is why we focus on state tracking instead of output tracking. To facilitate the design of controller, we first investigate the pole placement problem for SIMO systems using dynamic gains, when all the state variables are available for feedback. Both the local and global pole placement control has been accomplished in the MATLAB and SIMULINK environments. In both the cases the results are shown in the comparative mode between the demand and controlled states.

II. THE DESIGN PROBLEM

In this work, we consider the modeling to evolve a dynamic model-based control of the air jet impingement cooling of steel plate. Here we propose the design procedure based on state space pole placement techniques for the state feedback control design of jet impingement cooling system. The design of State-Space models is not different from that of transfer functions in that the differential equations describing the system dynamics are written first. For state-Space models, models, instead, the equations are arranged into a set of first order differential equations in terms of selected state variables, and the outputs are expressed these same state variables, Because the elimination of variables between equations is not an inherent part of this process, state models can be easier to obtain. The State models are directly derived from the original system equations. The standard form of the State-Space model equation is:

$$\dot{\mathbf{X}}(\mathbf{t}) = \hat{\mathbf{A}}\mathbf{X}(\mathbf{t}) + \hat{\mathbf{B}}\bar{a}(\mathbf{t}) \text{ (State Equation)}$$

Here 'x' is the State Vector, the vector of state variables, \bar{a} is the control variable and 'A' is the system coefficient matrix. The matrices $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are controllable. A fixed dynamic controller is designed i.e.,

$$\bar{a} = \hat{\mathbf{B}}^{-1}[\hat{\mathbf{A}}\bar{\mathbf{X}} - \dot{\bar{\mathbf{X}}}]$$

The pole placement design of 'B' in the form of full state feedback becomes equivalent to design 'K' where,

$$a_e = -\mathbf{K}\mathbf{X}_e \text{ (Controller or Control Law)}$$

(1)

For stability all the Eigen values of 'A' must be such that the real part is negative. Let us assume,

$$\mathbf{Z} = \mathbf{T}\mathbf{X} \tag{2}$$

The equation becomes,

$$\begin{aligned} \mathbf{T}\dot{\mathbf{X}} &= \mathbf{T}\mathbf{A}\mathbf{X} + \mathbf{T}\mathbf{B}\bar{a} \\ \dot{\mathbf{Z}} &= \mathbf{T}\mathbf{A}\mathbf{T}^{-1}\mathbf{Z} + \mathbf{T}\mathbf{B}\bar{a} \quad (\because \mathbf{X} = \mathbf{T}^{-1}\mathbf{Z}) \\ \dot{\mathbf{Z}} &= \bar{\mathbf{A}}\mathbf{Z} + \bar{\mathbf{B}}\bar{a}; \end{aligned} \tag{3}$$

Where $\bar{\mathbf{A}} = \mathbf{T}\mathbf{A}\mathbf{T}^{-1}$ and $\bar{\mathbf{B}} = \mathbf{T}\mathbf{B}$

Let us consider the Eigen values of equation (3) be $\bar{\lambda}$,

$$\text{Then, } |\bar{\mathbf{A}} - \bar{\lambda}\mathbf{I}| = 0$$

$$|\mathbf{A} - \bar{\lambda}\mathbf{I}| = 0 \Rightarrow \lambda = \bar{\lambda}$$

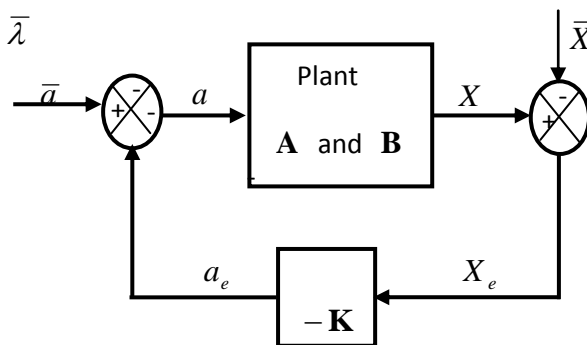


Figure 1: State Feedback Configuration

Since, $\bar{\lambda} = \lambda$, the transformation 'T' does not alter the system stability. The above diagram represents the corresponding control structure that evolves in the form of a feedback structure. In contrast to traditional feedback structure, **K** varies with time. Then feeding this input $\bar{\mathbf{a}}(\mathbf{t})$ back into the system, we obtained,

$$\dot{\mathbf{X}}(\mathbf{t}) = (\mathbf{A} - \mathbf{BK})\mathbf{X}(\mathbf{t}) \tag{4}$$

III. FULL STATE FEEDBACK DESIGN

There are several motivations for formulating the design procedure in state space. One motivation is a clear exposition of the controller structure. Another is that the controller structure points possibilities of reducing the order of the compensators using state space technique [26]. A third motivation is that the state space formulation may accommodate design procedures for systems in which the number of inputs or outputs is less than the number of operating conditions.

Let us consider the system P has $r > 1$ operating conditions with models $P_1(S), P_2(S), \dots, P_r(S)$. The state space representations of the models $P_i(S), i = 1, 2 \dots r$, are:

$$\dot{\mathbf{X}} = \mathbf{A}_i\mathbf{X} + \mathbf{B}_i\mathbf{a} \tag{5}$$

Where the state $\mathbf{X} \in \mathbf{R}^n$ and the input $\mathbf{a} \in \mathbf{R}^m$. The pairs $(\mathbf{A}_i, \mathbf{B}_i), i = 1, 2 \dots r$ are controllable. Let us design a fixed static controller $\mathbf{a} = -\mathbf{KX}$ such that the poles of the closed loop system are $\lambda(\mathbf{A}_i - \mathbf{B}_i\mathbf{K}) = \lambda_i, i = 1 \dots r$, where σ_i are the desired Eigen values for the i^{th} operating conditions. For full state to have a solution, it is necessary that the pairs $(\mathbf{A}_i, \mathbf{B}_i), i = 1 \dots r$, be controllable. Closely related to full state problem is the design of a fixed gain matrix **K** to stabilize all the operating conditions. The solution to problem exists a **K** such that:

$$\det(\mathbf{sI} - (\mathbf{A}_i - \mathbf{B}_i\mathbf{K})) = \mathbf{p}_i(\mathbf{s}), i = 1, \dots, r. \tag{6}$$

The nonlinear pole-placement equation (6) can be considered as Bass-Gura pole-placement formula [27], extended to multiple operating conditions.

IV. NUMERICAL METHODS FOR THE SINGLE POLE PLACEMENT PROBLEM

There are several methods for the pole placement problem some of the well known theoretical formulae, such as the Ackermann's formula, Bass-Gura formula etc.. The primary reason is that, these methods are based on transformation of the controllable pair $(\mathbf{A}_i, \mathbf{B}_i)$ to the controller companion form.

The computationally viable methods for pole placement are based on transformation of the pair $(\mathbf{A}_i, \mathbf{B}_i)$ to the controller Heisenberg form or the matrix **A** to the real Schur form (RSF), which can be achieved using orthogonal transformations. The pole placement design in the present study is accomplished in two ways, that is

- i) Assigning the fixed poles arbitrarily in the complex plane.
- ii) Assigning the variable poles in the complex plane.

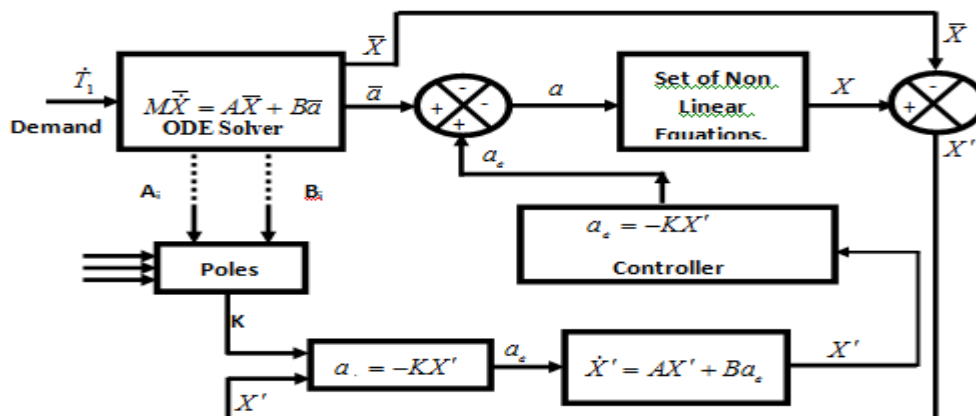


Figure 2: Block Diagram Showing Pole-Placement Control Model for Jet Impingement Cooling of Steel Plate (Temperature tracking Control).

V. STATE FEEDBACK CONTROL BY FIXED AND VARIABLE POLE PLACEMENT DESIGN

As a first attempt, a non-linear model-referenced state-feedback tracking control by fixed (local) pole placement for a jet impingement heat transfer system was implemented. The closed-loop poles were adjusted arbitrarily at each of the time intervals based on the open-loop poles of the system. The arbitrary pole assignment was performed in the SIMULINK environment. The poles were selected on the basis of trial and error method, for different simulation times. For each time segment the controlled output response and the reference output track were identified and compared. The results obtained in the SIMULINK environment for different time segments and time steps adopting the fixed pole placement control of the temperature track are discussed in detail in the later sections. After picking the feedback gain matrix \mathbf{K} to get the dynamics to have some nice properties (i.e., Stabilize A) $\lambda_i(\mathbf{A}) \rightarrow \lambda_i(\mathbf{A} - \mathbf{BK})$, the selected poles were placed in the complex plane. Recalling that the characteristic polynomial for this closed-loop system is the determinant of $(s\mathbf{I} - (\mathbf{A} - \mathbf{BK}))$. Since the matrices \mathbf{A} and $\mathbf{B}^*\mathbf{K}$ are both 3 by 3 matrices, there were 3 poles for the system. By using full state feedback we placed the poles at different places arbitrarily. The control matrix \mathbf{K} , which gave the desired poles, was found by using the MATLAB function **PLACE**. For SIMO systems, **PLACE** use the extra degrees of freedom to find a robust solution for \mathbf{K} i.e., minimizes sensitivity of closed loop poles to perturbations in \mathbf{A} and \mathbf{B} .

Then, the pole placement design procedures have been executed for the state feedback design of systems with single operating condition. To facilitate the design, poles varying with time were introduced. The same controller using strictly the state space technique was used for simulation i.e., $\dot{\mathbf{a}} = -\mathbf{KX}$. This approach provided a clear exposition of the controller structures. The closed-loop poles were adjusted at each of the time intervals based on the open-loop poles of the system. The feedback gain was developed based on the plant models and the corresponding set of closed-loop poles. The response of the non-linear plant to this controller was presented for the full state feedback. In order to target a steady state of the system the Eigen values of $(\mathbf{A} - \mathbf{BK})$ should be such that their real parts are negative. They were chosen in the vicinity of the open-loop poles of the system. If any of the open-loop poles have positive real part, the real part of the corresponding closed-loop poles were chosen to be zero. The time variant Eigen values of the matrix $(\mathbf{A} - \mathbf{BK})$ were computed then computed.

VI. RESULTS AND DISCUSSIONS

The main focus of this work was to evolve a model based closed loop control analysis with the application of an efficient pole placement technique. On the way to establish the necessary control model, we captured an important jet impingement cooling of a steel plate. The simulations were carried out in the MATLAB and SIMULINK environment. Different sets of results were generated for different set of poles. The results discussed below are both for fixed closed loop pole placement and variable pole placement problems. In the closed loop control phenomena, the important task was evaluation of elements of the feedback gain matrix (\mathbf{K}).

6.1 Closed- loop control results for fixed pole placement technique

is observed that with the application of fixed poles such as two complex conjugate poles and one real pole, where all the real poles are negative i.e., poles are considered more in the left half of the complex plane, the cooling rate controls upto -1 K/s and the nature of controlled temperature track is almost coincides over the reference temperature track as shown in the figures 3 and 4. The poles selected initially are $-0.2 \pm 1i$ (Complex Conjugate) and -100 (Real). Several trials were carried out by changing the pole locations and plots were generated for the temperature track on the top surface of the steel plate to realize the controlled cooling rate.

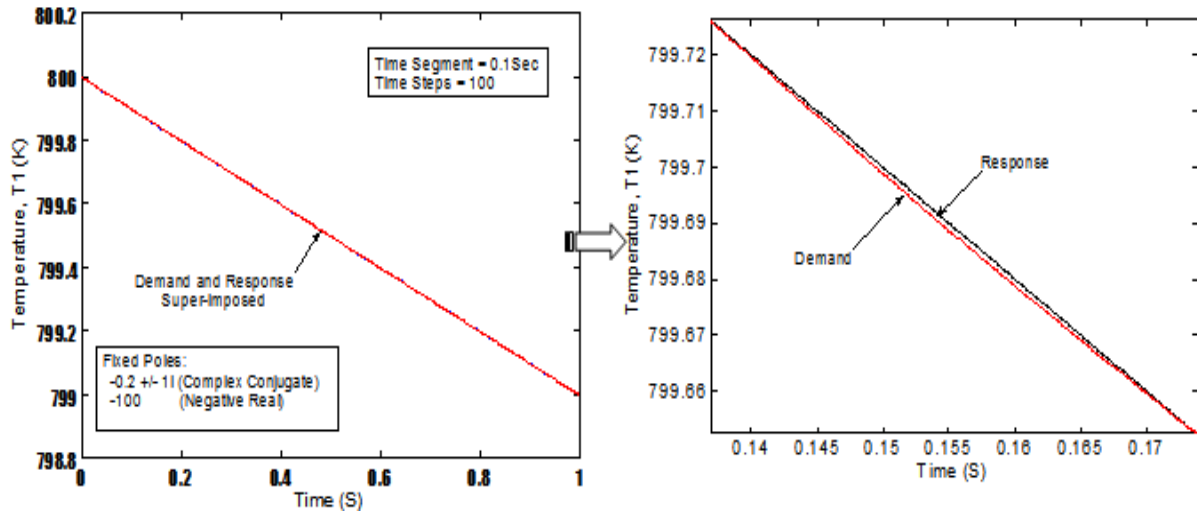


Figure 3: Effect of linearization time interval on tracking control of Plate temperature by assigning a fixed set of

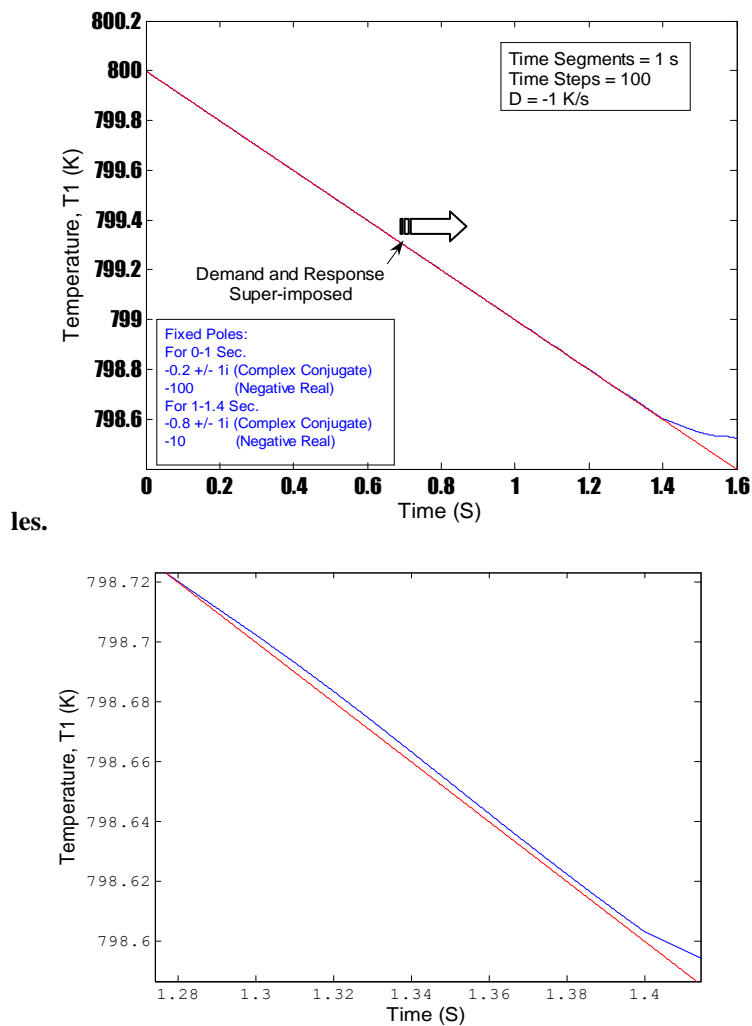


Figure 4: Effect of linearization time interval on tracking control of Plate temperature by assigning a fixed set of poles at Time Steps of 100 and Time segment of 1 second.

6.2 Closed- Loop control results for variable pole placement technique

This technique applies the Eigen values of the matrix (A) where, the matrix A is a 3 by 3 matrix, so there will be 3 poles for the system. Using the MATLAB function *eig*, the Eigen values of the (A) matrix are evaluated to evolve the feedback gain matrix (K). The problem was solved as a closed loop feedback control model using time dependent poles. For the positive values of the poles, the stability of the system was found marginal. Thus, at each linearized time segment the appropriate poles were assigned and the corresponding gain matrix was computed. The total computation has been performed by the help of SIMULINK tools. The effects of linearization time segment on tracking control of Plate temperature by assigning variable poles at different time locations are presented in the figures 5, 6, and 7 respectively. The only peculiarity found in this case was that, the response was oscillatory in nature. These results clearly predict that, one of the dominant poles tend to more negative side, while other two become less effective.

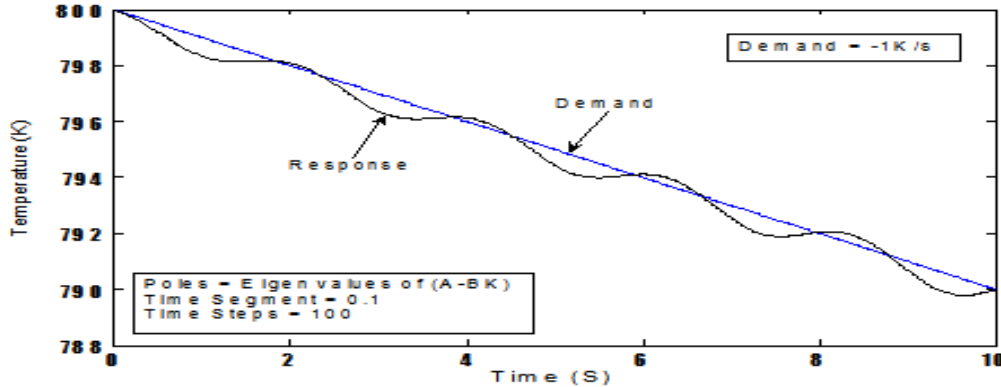


Figure 5: Effect of Linearization Time Interval on Tracking Control of Plate Temperature by Assigning a Set of Variable Poles at Different Time Locations at Time Steps of 100 and Time Segment of 0.1 second.

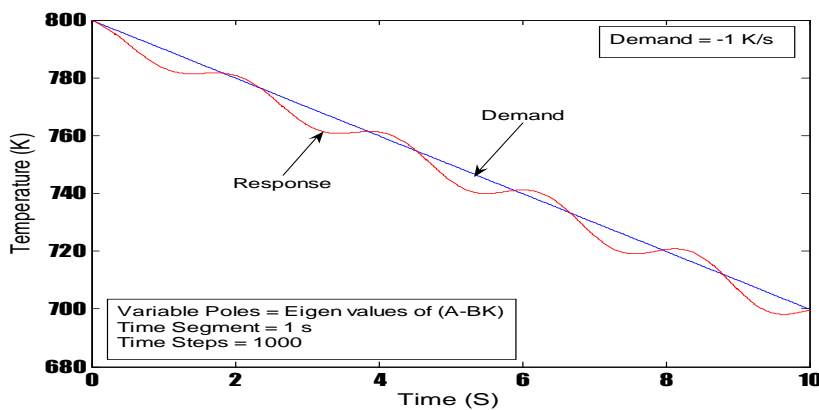


Figure 6: Effect of Linearization Time Interval on Tracking Control of Plate Temperature by Assigning a Set of Variable Poles at Different Time Locations at Time Steps of 1000 and Time Segment of 1 second.

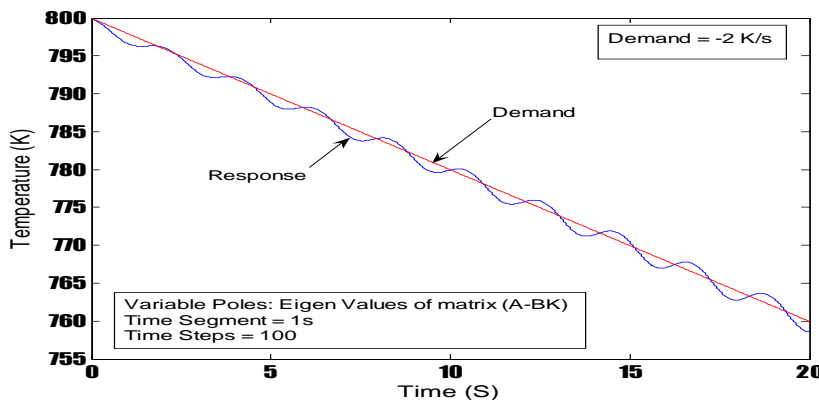


Figure 7: Effect of Linearization Time Interval on Tracking Control of Plate Temperature by Assigning a Set of Variable Poles at Different Time Locations at Time Steps of 100 and Time Segment of 1 second.

VII. CONCLUSION

A model based simulation and control of the temperature track in jet impingement cooling process of a steel plate has been developed that incorporates the implementation of an efficient pole placement technique in two ways i.e., one by the local poles and other by the global poles. The nonlinear control model developed here can be used to evaluate the performance in terms of physical properties and metallurgical aspects. The conclusions being obtained from the work are:

1. Pole placement design procedures have been proposed for the state feedback design of systems with single input condition. To facilitate the design, the concepts of coefficient matrices (A and B) and local poles are introduced.
2. It is seen that the response of the non-linear system is sensitive to linearization time interval. Better control is implemented by increasing the frequency of adjustment of the closed-loop poles.
3. In the present algorithm, the mean solution is utilized at discrete intervals (Piece-wise linearized time intervals) of time for evaluation of the coefficients.

It is observed from both the fixed pole placement and variable pole placement that, in case of fixed pole placement, the simulation time is limited upto 1.4 seconds. Within this simulation time, the demand temperature track and the response are coinciding to each other with very negligible error. While, in case of variable pole placement, the simulation time is large for the same linearized time intervals and percentage of error is significant compared to the first case.

REFERENCES

- [1]. D.Q. Mayne, J.B. Rawlings, C.V. Rao, P.O.M. Scokaert. constrained model predictive control: Stability and Optimality. *Automatica* 2000, 36 (6), 789.
- [2]. F. Allgower, A. Zheng. *Nonlinear Model Predictive Control; Progress in Systems and Control Theory Series*; Birkhauser Verlag; Basel 2000; Vol. 26.
- [3]. S.L. de Oliveira, M.V. Kothare, M. Morari. Contractive model predictive control for constrained nonlinear systems. *IEEE Trans. Autom. Control* 2000, 45, 1053.
- [4]. C. Kravaris, P. Daoutidis, R.A. Wright. Output feedback control of no minimum-phase nonlinear processes. *Chem. Engg. Sc.*, 1994, 49 (13), 2107.
- [5]. A. Isidori. *Nonlinear control systems*; Springer- Verlag; New York, 1995.
- [6]. N.H. El-Farra, P.D. Christofides. Integrating Robustness, Optimality and constraints in control of Non linear processes. *Chem. Engg. Sci.* 2001, 56, 1841.
- [7]. N.H. El-Farra, P.D. Christofides. Bounded Robust control of constrained nonlinear processes. *Chem. Engg. Sc.* 2003, 58, 3025.
- [8]. S. Djuljevic, N. Kazantzis. A New Lyapunov design approach for nonlinear systems based on Zubov's method. *Automatica* 2002, 38, 1999.
- [9]. C. Kravaris, P. Daotidis. Nonlinear State feedback control of second order non-minimum phase nonlinear systems. *Comput. Chem. Eng.* 1990, 40 (4/5), 439.
- [10]. M. Niemiec, C. Kravaris. Nonlinear model-state feedback control for non-minimum -phase processes. *Automatica* 2003, 39, 1295.
- [11]. J.M. Kanter, M. Soroush, W.D. Seider. Nonlinear controller design for input-constrained, multivariable processes. *Ind. Eng. Chem. Res.* 2002, 41, 3735.
- [12]. C. Kravaris, M. Niemiec, N. Kazantzis. Singular PDEs and the assignment of zero dynamics in nonlinear systems. *Syst. Control Lett.* 2004, 51, 67.
- [13]. M.C. Mickle, R. Huang, J.J. Zhu. Unstable, non-minimum phase, nonlinear tracking by trajectory linearization control. *Proceedings of the IEEE International Conference on Control applications, Taipei, Taiwan, 2004*; p812.
- [14]. C.T. Tomlin, S.S. Sastry. Bounded tracking for non-minimum phase nonlinear systems with fast zero dynamics. *Int. J. Control* 1997, 68, 819.
- [15]. A.J. Van der Schaft. L_2 - Gain analysis of nonlinear systems and nonlinear state feedback H_∞ control. *IEEE Trans. Autom. Control* 1992, 37, 770.
- [16]. S. Devasia, D. Chen, B. Paden. Nonlinear inversion-based output tracking. *IEEE Trans. Autom. Control* 1996, 41, 930.
- [17]. S. Devasia. Approximated stable inversion for nonlinear systems with non-hyperbolic internal dynamics. *IEEE Trans. Autom. Control* 1999, 44, 1419.
- [18]. L.R. Hunt, G. Meyer. Stable inversion for nonlinear systems. *Automatica* 1997, 33, 1549.
- [19]. A. Isidori, C.I. Byrnes. Output regulation of nonlinear systems. *IEEE Trans. Autom. Control* 1990, 35, 131.
- [20]. D.G. Chen, B. Paden. Stable inversion of nonlinear non-minimum phase systems. *Int. J. Control* 1996, 64, 81.
- [21]. J. Ackermann. (1993). *Robust Control: Systems with Uncertain physical parameters*. Springer-Verlag, New York, NY.
- [22]. B.R. Barmish. (1994). *New tools for robustness of linear systems*. Macmillan Publishing Co., New York, NY.
- [23]. S.P. Bhattacharya, H. Chapellat, L.H. Keel. (1995). *Robust Control- The parametric approach*. Prentice Hall publishing Co, Upper Saddle River, NJ.
- [24]. Y.C. Soh, R.J. Evans, I. Petersen, R.E. Betz. Robust Pole assignment. *Automatica*, 1987, 23, pages 601-610.
- [25]. L.H. Keel, S.P. Bhattacharya. A linear programming approach to controller design. *Proceedings of the 36th Conference on Decision and control.* 1997, pages 2139-2148, San Diego, CA, USA.
- [26]. T. Kailath, *Linear systems*, Anglewood Cliffs, Prentice-Hall, 1980.
- [27]. R. W. Bass, I. Gura, "High order design via State space considerations", *Proc. 1965 JACC*, pp. 311-318.
- [28]. Chow, J.H. A pole placement design approach for systems with multiple operating conditions, *Proc. 27th Conf. On Decision and Control, Austin, Texas.* 1988, pages 1272-1277.